DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose, but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.
1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word line indicates a straight line.
4. The word average indicates arithmetic mean.

1. The cost, in dollars, to paint a room that has an area to be painted of $A$ square feet is $0.75A + 20h$, where $h$ is the number of hours it takes to paint the room. What is the cost of painting a room that has an area to be painted of 120 square feet and takes 2 hours to paint?
   A. $120
   B. $122
   C. $130
   D. $160
   E. $220

2. What is the least common denominator of the fractions $\frac{4}{15}$, $\frac{1}{6}$, and $\frac{3}{4}$?
   F. 20
   G. 60
   H. 90
   J. 120
   K. 360

3. Malik is building a frame for a rectangular picture that he painted, and he needs to know the perimeter of the picture. The length of the picture is 36 inches and the width is 24 inches. What is the perimeter, in inches, of the picture?
   A. 60
   B. 84
   C. 96
   D. 120
   E. 864

4. In $\triangle ABC$, the sum of the measures of $\angle A$ and $\angle B$ is 64°. What is the measure of $\angle C$?
   F. 26°
   G. 52°
   H. 64°
   J. 116°
   K. 128°
5. On a map, $\frac{1}{4}$ inch represents 10 miles. How many inches on this map represent 250 miles?
   A. $2\frac{1}{2}$
   B. $6\frac{1}{4}$
   C. 25
   D. 40
   E. $62\frac{1}{2}$

6. The 2 cubes shown below have diagonals $AB$ and $CD$, respectively. The side lengths given are in feet. What is the ratio of the length of $AB$ to the length of $CD$?

   F. 1:4
   G. 1:16
   H. 4:1
   J. 16:1
   K. 64:1

7. What is the value of $-x + y + z$ for $x = -1$, $y = -3$, and $z = 2$?
   A. -6
   B. -2
   C. 0
   D. 4
   E. 6

8. If $9 + 3x = 27$, then $2x = ?$
   F. 6
   G. 12
   H. 13
   J. 18
   K. 24

9. A woman purchased 100 shares of stock at $5.00 per share. If each share rose $0.10 the first month, decreased $0.08 the second month, and gained $0.03 the third month, what is the value of the woman's investment?
   A. $505
   B. $520
   C. $525
   D. $1,505
   E. $1,545
10. At Acme Manufacturing Company, each employee's annual salary for next year will be $3 \frac{1}{2}$% more than this year's annual salary. An employee whose annual salary this year is $32,000.00 will have what annual salary next year?

F. $32,003.50
G. $32,035.00
H. $32,350.00
J. $33,120.00
K. $41,600.00

11. Vat 1, Vat 2, and Vat 3, when full, each hold the same amount of water. At the present time, Vat 1 is $\frac{5}{6}$ full, Vat 2 is $\frac{1}{12}$ full, and Vat 3 is $\frac{1}{3}$ full. Water will be transferred between the vats so that each of the 3 vats contains the same amount of water. After the transfer, each of the 3 vats will be what fraction full?

A. $\frac{1}{3}$
B. $\frac{1}{6}$
C. $\frac{1}{9}$
D. $\frac{5}{8}$
E. $\frac{5}{12}$

12. Which of the following graphs shows the solution set for the inequality $4x - 2 \geq 6$?

F. 

G. 

H. 

J. 

K. 

GO ON TO THE NEXT PAGE.
13. The Hopc-A-Lot Foundation is mailing brochures to 4,000 prospective donors. The foundation’s goal is to have proceeds of $1,500 after paying $900 for the mailing. According to past mailings, the average donation was $20 per donor. Assuming this average, how many of the prospective donors need to donate to reach the goal?
A. 30
B. 45
C. 120
D. 200
E. 1,500

14. Lines $\overrightarrow{BC}$ and $\overrightarrow{DE}$ are parallel, and transversals $\overrightarrow{BD}$ and $\overrightarrow{CE}$ intersect at $A$, as shown in the figure below.
Given that $\triangle ABC$ is an equilateral triangle, $x = ?$

F. 30
G. 60
H. 120
J. 130
K. 150

15. What is the positive solution to the equation $16x^2 = 30$?
A. $\frac{30}{16}$
B. $\left(\frac{30}{16}\right)^2$
C. $\sqrt{\frac{150}{30}}$
D. $\sqrt{\frac{16}{30}}$
E. $\sqrt{\frac{30}{16}}$

16. If Kusum uses 1 pound of grass seed per 800 square feet to be seeded, how many pounds of grass seed will she use to seed the region shown below?
F. $3\frac{1}{4}$
G. 4
H. $4\frac{1}{4}$
J. $4\frac{3}{4}$
K. 5
An organization promoting good nutritional habits collected data on fat calories in foods from 9 fast-food restaurants in Mesa City. The values in the list below represent the number of fat calories in a small order of french fries at each of these fast-food restaurants.


17. Based on the data listed, what is the median number of fat calories in a small order of french fries at these 9 restaurants?
   A. 106
   B. 108
   C. 125
   D. 128
   E. 160

18. Which of the following bar graphs most accurately represents the data on the number of fat calories in a small order of french fries at the 9 fast-food restaurants?

   F.  
   G.  
   H.  
   J.  
   K.  

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19. Of the 9 fast-food restaurants, Hungry Henry's has the lowest number of fat calories in a small order of french fries. At Henry's, 43% of the total number of calories in french fries are fat calories. Which of the following values is closest to the total number of calories in a small order of Henry's french fries?

A. 127
B. 132
C. 139
D. 207
E. 223

20. The organization collects data from 2 additional restaurants and includes the new data in the list. The number of fat calories in a small order of french fries at each of the 2 additional restaurants is designated by \( x \) and \( y \), respectively. Which of the following expressions gives the average of this larger list of values?

F. \( \frac{932 + x + y}{10} \)
G. \( \frac{932 + x + y}{11} \)
H. \( \frac{1.092 + x + y}{9} \)
J. \( \frac{1.092 + x + y}{9 + x + y} \)
K. \( \frac{1.092 + x + y}{11} \)

21. A bag contains 8 red marbles, 5 yellow marbles, and 11 green marbles. How many additional red marbles must be added to the 24 marbles already in the bag so that the probability of randomly drawing a red marble is \( \frac{3}{5} \)?

A. 11
B. 16
C. 20
D. 24
E. 32

22. The sum of \( (3x^3 + 4x^2 - 3x + 1) \) and which of the following polynomials results in the polynomial \( (5x^3 - 4x^2 + 7x - 3) \)?

F. \(-2x^3 + 8x^2 - 10x + 4\)
G. \(2x^3 - 8x^2 + 10x - 4\)
H. \(2x^3 + 10x + 2\)
J. \(8x^3 + 4x - 2\)
K. \(15x^3 - 16x^2 - 21x - 3\)
23. The table below gives the price to mail a single package through the United States Postal Service on August 30, 2007. The price depended on the weight of the package.

<table>
<thead>
<tr>
<th>Weight in pounds</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; x ≤ 2</td>
<td>$4.60</td>
</tr>
<tr>
<td>2 &lt; x ≤ 3</td>
<td>$5.05</td>
</tr>
<tr>
<td>3 &lt; x ≤ 4</td>
<td>$5.70</td>
</tr>
<tr>
<td>4 &lt; x ≤ 5</td>
<td>$6.30</td>
</tr>
</tbody>
</table>

Which of the following graphs best represents this information?

A.  
B.  
C.  
D.  
E.  

24. What is the value of \( f(-4) \) given \( f(x) = 5x^2 - 2x + 10 \)?

F.  \(-382\)  
G.  \(-62\)  
H.  \(82\)  
J.  \(98\)  
K.  \(418\)

25. All of the following monomials are factors, over the integers, of \( 18x^2y + 12x^3y^3 - 6x^3y \) EXCEPT:

A.  \(6\)  
B.  \(3x\)  
C.  \(2x^2\)  
D.  \(12x^2\)  
E.  \(x^3y\)
26. Josh is standing in a pool and looking up at his friend Olivia. Olivia is lying on her stomach on the diving board looking at Josh. The horizontal and vertical distances, in meters, between Josh and Olivia are given in the diagram below. What is the measure of the angle of elevation, \( \theta \), of Josh's line of sight?
(Note: Drawing is NOT to scale.)

![Diagram of a pool and a diving board with a person looking up at another person standing in the pool.]

F. \( \text{Arcsin} \left( \frac{3}{8} \right) \)
G. \( \text{Arccos} \left( \frac{3}{8} \right) \)
H. \( \text{Arctan} \left( \frac{3}{8} \right) \)
J. \( \text{Arccot} \left( \frac{3}{8} \right) \)
K. \( \text{Arccsc} \left( \frac{3}{8} \right) \)

27. Avari traveled the 2-mile trail from her house to Big Lake on her bicycle. She then traveled 3 times around the Big Lake Loop and returned home by the 2-mile trail. At the end of her bicycle ride, the trip odometer showed that she had traveled 22 miles. Which of the following equations, when solved, gives the distance Avari traveled once around Big Lake Loop, \( d \) miles?

A. \( 2 + d = 22 \)
B. \( 2 + 3d = 22 \)
C. \( 4 - 3d = 22 \)
D. \( 4 + d = 22 \)
E. \( 4 + 3d = 22 \)

28. In the standard \((x,y)\) coordinate plane, what is the slope of the line with equation \( 3x + 5y = 6 \)?

F. \( -\frac{5}{3} \)
G. \( -\frac{3}{5} \)
H. \( \frac{3}{5} \)
J. \( \frac{5}{3} \)
K. \( 2 \)
29. What is the solution to the equation below?

\[ 3(x - 4) - 2(x - 3) = 5(-x - 3) + 6 \]

A. \(-\frac{5}{2}\)
B. \(-\frac{3}{2}\)
C. \(-\frac{1}{2}\)
D. \(\frac{3}{2}\)
E. \(\frac{9}{2}\)

30. If \(0 < x < 1\), and \(k\) is a positive integer, then what must be true about the value of \(x^k\)?

F. \(x^k < -1\)
G. \(-1 < x^k < 0\)
H. \(0 < x^k < 1\)
J. \(x^k > 1\)
K. \(x^k = 0\)

31. For all \(a \neq 0\), \(\frac{(2a^2)^3 + 3a^4 - 5a^2}{2a} = ?\)

A. \(3a - a^3\)
B. \(4a^4 - a^3\)
C. \(3a^5 - a^3\)
D. \(4a^3 - 11a^3\)
E. \(4a^3 - a^3\)

32. The diameter of Earth is about \(1.28 \times 10^4\) km. The diameter of the Moon is about \(3.5 \times 10^3\) km. Which of the following is closest to the difference, in kilometers, between the diameter of Earth and the diameter of the Moon?

F. \(2.2 \times 10^3\)
G. \(2.2 \times 10^4\)
H. \(9.3 \times 10^3\)
J. \(9.3 \times 10^2\)
K. \(9.3 \times 10^4\)

33. In the right triangle shown below, the length of \(AC\) is 4 mm and the length of \(BC\) is 3 mm. For \(\angle A\), the value of which of the following trigonometric expressions is \(\frac{2}{3}\)?

A. \(\sin A\)
B. \(\cos A\)
C. \(\tan A\)
D. \(\csc A\)
E. \(\cot A\)
34. When light shines on an object, the intensity of that light, \( I \), in units of light intensity, can be expressed as \( \frac{k}{d^2} \), where \( d \) is the distance, in feet, the light source is from the object, and \( k \) is a proportionality constant. For one light source shining on an object, \( d = 12 \) and \( I = 10 \). If \( d = 6 \) for the same light source shining on the same object, what is the corresponding value of \( I \)?

F. 20  
G. 33 \( \frac{1}{3} \)  
H. 40  
J. 200  
K. 1,440

35. A circle in the standard (x,y) coordinate plane has its center at \((-2,4)\) and passes through \((3,16)\). What is the area, in square coordinate units, of this circle?

A. \( 26\pi \)  
B. \( 34\pi \)  
C. \( 169\pi \)  
D. \( 289\pi \)  
E. \( 441\pi \)

36. Royce plans to construct a triangular flower bed on the corner of his property where a sidewalk forms a right angle. The flower bed and the lengths, in feet, of 2 of its sides are shown in the figure below. The flower bed will be enclosed by a garden fence that is set up along its entire perimeter. To the nearest foot, how many feet of garden fence will enclose the flower bed?

F. 12  
G. 14  
H. 16  
J. 17  
K. 20

37. If \( 2w + 7 = |\ -2\ | \), how many different values are possible for \( w \)?

A. 0  
B. 1  
C. 2  
D. 3  
E. Infinitely many
38. The graph of the parabola with equation \( y = x^2 + 2x - 8 \) is shown in the standard \((x,y)\) coordinate plane below. One of the following graphs is the graph of the reflection of the parabola over the \(y\)-axis. Which one is it?

F. 

G. 

H. 

J. 

K.

39. The points \(A(12,18)\) and \(B(-4,2)\) lie in the standard \((x,y)\) coordinate plane. What are the coordinates of the midpoint of \(AB\)?

A. \((4, 8)\)

B. \((4,10)\)

C. \((8, 8)\)

D. \((8, 20)\)

E. \((16,16)\)
40. For \( \triangle ABC \) shown below, the length of \( BC \) is 50 mm. Which of the following equations, when solved, will give the length, in millimeters, of \( AB \)?

(Note: The law of sines states that given \( \triangle XYZ \),
\[
\frac{\sin \angle X}{YZ} = \frac{\sin \angle Y}{XZ} = \frac{\sin \angle Z}{XY}.
\]

F. \( \frac{\sin 68^\circ}{50} = \frac{\sin 58^\circ}{AB} \)
G. \( \frac{\sin 58^\circ}{50} = \frac{\sin 68^\circ}{AB} \)
H. \( \frac{\sin 58^\circ}{50} = \frac{\sin 54^\circ}{AB} \)
J. \( \frac{\sin 54^\circ}{50} = \frac{\sin 68^\circ}{AB} \)
K. \( \frac{\sin 54^\circ}{50} = \frac{\sin 58^\circ}{AB} \)

41. The vertices of \( \triangle AOB \) are \( A(0,6) \), \( O(0,0) \), and \( B(8,0) \), as shown in the standard \((x,y)\) coordinate plane below. What are the coordinates of the center of the circle that circumscribes \( \triangle AOB \)?

A. \( (0,3) \)
B. \( (2,1,5) \)
C. \( (4,0) \)
D. \( (4,3) \)
E. \( (7,7) \)

42. Circle \( P \) has a radius of 4 units and is in the standard \((x,y)\) coordinate plane. The set of all points in the coordinate plane that are 3 units from the center of Circle \( P \) is a circle that:

F. intersects Circle \( P \) at 2 points.
G. is internally tangent to Circle \( P \).
H. is externally tangent to Circle \( P \).
J. is interior to and does not intersect Circle \( P \).
K. is exterior to and does not intersect Circle \( P \).

43. Suppose that \( x \) is a positive real number and \( \frac{4x}{6x^2} \) is a rational number. Which of the following statements about \( x \) must be true?

A. \( x \) is rational
B. \( x \) is irrational
C. \( x = 1 \)
D. \( x = \frac{2}{3} \)
E. \( x = \frac{3}{2} \)
44. The functions \( f \) and \( g \) are defined as \( f(x) = 3x + 2 \) and \( g(x) = 2x - 1 \). What is the value of \( f(g(-2)) \)?

F. 20  
G. -4  
H. -5  
J. -9  
K. -13

45. Consider all rectangles with an area of 36 square feet. \( P \) is the perimeter, in feet, of at least 1 of these rectangles if and only if:

A. \( P \geq 6 \)  
B. \( P \geq 24 \)  
C. \( P \geq 36 \)  
D. \( P \geq 144 \)  
E. \( P \geq 324 \)

46. Carmen drove from Blairtown to Ore City, a distance of 80 miles. From Ore City she drove on to Janesville, and then drove back to Blairtown. The ratio of Carmen's driving times on the first, second, and third segments of the trip, respectively, was 5:2:4, and she drove at the same average speed on each segment. What was Carmen's total driving distance, in miles, for the 3 segments of the trip?

F. 176  
G. 220  
H. 240  
J. 360  
K. 440

47. In \( \triangle ABC \), the measure of \( \angle A \) is 43° and the measure of \( \angle C \) is 32°. Which of the following inequalities involving the lengths of the sides of \( \triangle ABC \) is true?

A. \( AB > AC \)  
B. \( AB > BC \)  
C. \( AC > BC \)  
D. \( AC > AB + BC \)  
E. \( BC > AC \)

48. One of the following functions is graphed in the standard \((x,y)\) coordinate plane below. Which function is it?

F. \( y = |x - 1| - 2 \)  
G. \( y = |x + 1| - 2 \)  
H. \( y = |x + 1| + 2 \)  
J. \( y = |x - 2| + 1 \)  
K. \( y = |x + 2| - 1 \)
49. One of the following is an equation of the ellipse shown in the standard \((x,y)\) coordinate plane below. Which one?

(Note: The coordinate unit on the \(x\)-axis is the same length as the coordinate unit on the \(y\)-axis.)

\[
\begin{align*}
A. & \quad x^2 - y^2 = 16 \\
B. & \quad (x - 8)^2 + (y - 4)^2 = 16 \\
C. & \quad \frac{(x + 8)^2}{4} - \frac{(y + 4)^2}{2} = 1 \\
D. & \quad \frac{(x + 8)^2}{16} + \frac{(y + 4)^2}{4} = 1 \\
E. & \quad \frac{(x - 8)^2}{16} + \frac{(y - 4)^2}{4} = 1
\end{align*}
\]

50. A regular pyramid with a square base is shown in the figure below. The slant height is \(\sqrt{3}\) units and the length of the base edge is 2 units. What is the total length, in units, of all 8 edges of the pyramid?

\[
\text{F. } 4\sqrt{7} \\
\text{G. } 4\sqrt{7} + 8 \\
\text{H. } 8 \\
\text{J. } 14 \\
\text{K. } 16
\]

51. The solution of the system of equations below is the set of all \((x,y)\) such that \(2x - 3y = 6\). What is the value of \(k\)?

\[
\begin{align*}
18x - 27y &= 54 \\
6x + ky &= -2k
\end{align*}
\]

\[
\text{A. } -9 \\
\text{B. } -1 \\
\text{C. } 3 \\
\text{D. } 6 \\
\text{E. } 9
\]

52. The solution set for the equation \(2x^2 + 1 = 1\) contains:

\[
\text{F. } 2 \text{ imaginary numbers.} \\
\text{G. } 2 \text{ positive real numbers.} \\
\text{H. } 1 \text{ negative and } 1 \text{ positive real number.} \\
\text{J. } 1 \text{ negative real number only.} \\
\text{K. } 1 \text{ real number, which is 0.}
\]

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Use the following information to answer questions 53–55.

In the figure below, a large circle with center $O$ has a diameter $AB$ that is 40 mm long. Point $C$ lies on the large circle such that the measure of $\angle ABC$ is $60^\circ$. A diameter of the small circle is $\overline{AO}$.

![Diagram with points A, B, C, and O labeled]

53. What is the area, in square millimeters, of the small circle?
   A. $10\pi$
   B. $20\pi$
   C. $40\pi$
   D. $100\pi$
   E. $400\pi$

54. What is the length, in millimeters, of arc $\overline{AB}$?
   F. 20
   G. $20\pi$
   H. 40
   J. $40\pi$
   K. 80

55. The figure is placed in the standard $(x,y)$ coordinate plane so that $A$ has coordinates $(-20,0)$ and $B$ has coordinates $(20,0)$. What is the $x$-coordinate of $C$?
   A. $-15$
   B. $-10$
   C. 0
   D. 10
   E. 15

56. In the figure shown below, $C, M,$ and $N$ lie on the circle whose center is $O$, and $\angle MON$ is a right angle. What is the sum of the measures of $\angle CMO$ and $\angle CNO$?
   F. $90^\circ$
   G. $67.5^\circ$
   H. $60^\circ$
   J. $45^\circ$
   K. $22.5^\circ$
57. The length of the shorter side of rectangle $ABCD$ is 4 inches less than the length, $L$, of the longer side. The length of the longer side of rectangle $WXYZ$, which is similar to $ABCD$, is $10L$ inches. In terms of $L$, what is the length of the shorter side of $WXYZ$?

A. $L - 40$
B. $L + 6$
C. $10L - 4$
D. $10L + 6$
E. $10L - 40$

58. For any integer $n > 0$, the triangular number $T_n$ is the number of dots in a triangular array with $n$ points on each side. The figure below shows the first 4 triangular numbers. What is the value of $T_{64}$?

\[ T_1 = 1 \quad T_2 = 3 \quad T_3 = 6 \quad T_4 = 10 \]

F. 189
G. 192
H. 2,016
J. 2,048
K. 2,080

59. For what integer $k$ are both solutions of the equation $x^2 + kx + 17 = 0$ positive integers?

A. -18
B. -16
C. 1
D. 16
E. 18

60. In $\triangle XYZ$, the measure of $\angle X$ is $90^\circ$, the measure of $\angle Z$ is $\theta$, $XY = 12$ units, and $\tan \theta = \frac{4}{9}$. What is the area of $\triangle XYZ$, in square units?

F. 162
G. 324
H. $2\sqrt{65}$
J. $6\sqrt{585}$
K. $12\sqrt{585}$